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Estimation of the order of integration in the UK and the us interest rates using fractionally integrated semiparametric techniques

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Abstract

We examine in this article the monthly structure of the US and the UK interest rates by means of using fractionally integrated semiparametric techniques. The results based on the quasi maximum likelihood estimate of Robinson (QMLE, 1995) indicate that the order of integration of both series is higher than 1, especially for the US, with the degree of integration oscillating around 1.23. For the UK, this value is around 1.07. Similar results are obtained when using a parametric testing procedure of Robinson (1994), though with this method, the unit root null hypothesis cannot be rejected for the UK. In conclusion, both series are nonstationary and non-mean-reverting.

Keywords:

Long memory; Fractional integration; Interest rates.
JEL *Classification*: C22.

1. Introduction

We are concerned in this article with the estimation of the fractional differencing parameter in the US and the UK monthly interest rates. The estimation of this parameter is important since it can give us some indication about the degree of persistence in the series. Traditionally, it has been assumed that the interest rates are I(1) and first differences have become a standard practice when modelling these series, especially in the context of cointegration. However, the unit root model is merely a particular case of a much more general class of long memory processes.

For the purpose of the present paper, we define an I(0) process, $\{u_t, t = 0, \pm 1, ...\}$, as a covariance stationary process with spectral density function that is positive and finite at the zero frequency.[1] In this context, we say that x_t is I(d) if

$$(1 - L)^d x_t = u_t, \qquad t = 1, 2, ... \tag{1}$$

$$x_t = 0, \qquad t \leq 0, \tag{2}$$

where L is the lag operator ($Lx_t = x_{t-1}$)[2] and where the unit root case corresponds to d = 1. If d > 0 in (1), x_t is said to be a long memory process, so-called because of the strong degree of correlation between observations widely separated in time. If d ∈ (0, 0.5), x_t is covariance stationary, and if d ∈ [0.5, 1), x_t is no longer stationary but it is still mean reverting, with the effects of the shocks dying away in the long run. Finally, if d ≥ 1, the series is nonstationary and non-mean-reverting. These processes were introduced by Granger and Joyeux (1980), Granger (1980, 1981) and Hosking (1981), (though earlier work by Adenstedt, 1974, and Taqqu, 1975, show an awareness of its representation), and were theoretically justified by Robinson (1978), Granger (1980), and more recently by Parke (1999). Empirical applications of fractional models like (1) on macroeconomic time series are amongst others the papers of Diebold and Rudebusch (1989), Baillie and Bollerslev (1994) and Gil-Alana and Robinson (1997).

There exist many approaches for estimating and testing the fractional differencing parameter. Some of them are parametric, in which the model is specified up to a finite number of parameters, (eg, Fox and Taqqu, 1986; Dahlhaus, 1989; Sowell, 1992; Robinson, 1994a; etc.). However, on estimating with parametric approaches, the correct choice of the model is important. If it is misspecified, the estimates are liable to be inconsistent. In fact, misspecification of the short-run components of the series may invalidate the estimation of the long run parameter. Thus, there may be some advantages on estimating d with semiparametric techniques. In this article, we propose the use of the quasi maximum likelihood estimate of Robinson (1995a). This

[1] A more general definition of I(0) processes is the one that assumes that the spectral density function is positive and finite at any frequency. However, here, we restrict ourselves to the case of the zero frequency.

[2] Equation (2) is a standard assumption which is usually made when modelling long memory time series. (See, e.g., Gil-Alana and Robinson, 1997).

procedure will be briefly described in Section 2. In Section 3, it will be applied to the US and the UK interest rate series while Section 4 contains some concluding comments.

2. The quasi maximum likelihood estimate of Robinson (1995a)

The quasi maximum likelihood estimate (QMLE) of Robinson (1995a) is basically a 'Whittle estimate' in the frequency domain, considering a band of frequencies that degenerates to zero. The estimate is implicitly defined by:

$$\hat{d} = \arg\, min_d \left(\log \overline{C(d)} - 2\, d \, \frac{1}{m} \sum_{j=1}^{m} \log \lambda_j \right), \tag{3}$$

$$\overline{C(d)} = \frac{1}{m} \sum_{j=1}^{m} I(\lambda_j)\, \lambda_j^{2d}, \qquad \lambda_j = \frac{2\,\pi\, j}{T}, \qquad \frac{m}{T} \rightarrow 0,$$

where $I(\lambda_j)$ is the periodogram of the raw time series, x_t, given by:

$$I(\lambda_j) = \frac{1}{2\pi T} \left| \sum_{t=1}^{T} x_t\, e^{i\lambda_j t} \right|^2 .$$

and $d \in (-0.5, 0.5)$.[2] Under finiteness of the fourth moment and other mild conditions, Robinson (1995a) proved that

$$\sqrt{m}\, (\hat{d} - d_o) \rightarrow_d N(0, 1/4) \quad as\ T \rightarrow \infty,$$

where d_o is the true value of d and with the only additional requirement that $m \rightarrow \infty$ slower than T. Robinson (1995a) proposes that m should be smaller than T/2. A multivariate extension of this estimation procedure can be found in Lobato (1999). There also exist other semiparametric procedures for estimating the fractional differencing parameter, for example, the log-

[2] Velasco (1999a, b) has recently showed that the fractionally differencing parameter can also be consistently semiparametrically estimated in nonstationary contexts by means of tapering.

periodogram regression estimate (LPE), initially proposed by Geweke and Porter-Hudak (1983) and modified later by Künsch (1986) and Robinson (1995b) and the averaged periodogram estimate (APE) of Robinson (1994b). However, we have decided to use in this article the QMLE firstly because of its computational simplicity. Note that using the QMLE, we do not need to employ any additional user-chosen numbers in the estimation (as is the case with the LPE and the APE). Also, we do not have to assume Gaussianity in order to obtain an asymptotic normal distribution, the QMLE being more efficient than the LPE. In addition, several Monte Carlo experiments carried out, for example, by Gil-Alana (2002) showed that, in finite samples (e.g., with $T < 300$) the QMLE has better statistical properties compared with the other procedures.

3. The order of integration in the UK and the US interest rates

The time series data analysed in this section correspond to the monthly observations of the UK and the US interest rates for the time period 1973.3 – 1990.5, obtained from Creedy et al. (1996). In that paper they use these variables as fundamentals to explain the dynamics of the exchange rates. For the US, the interest rate is the Federal Funds rate, while for the UK it is London Interbank Offer.

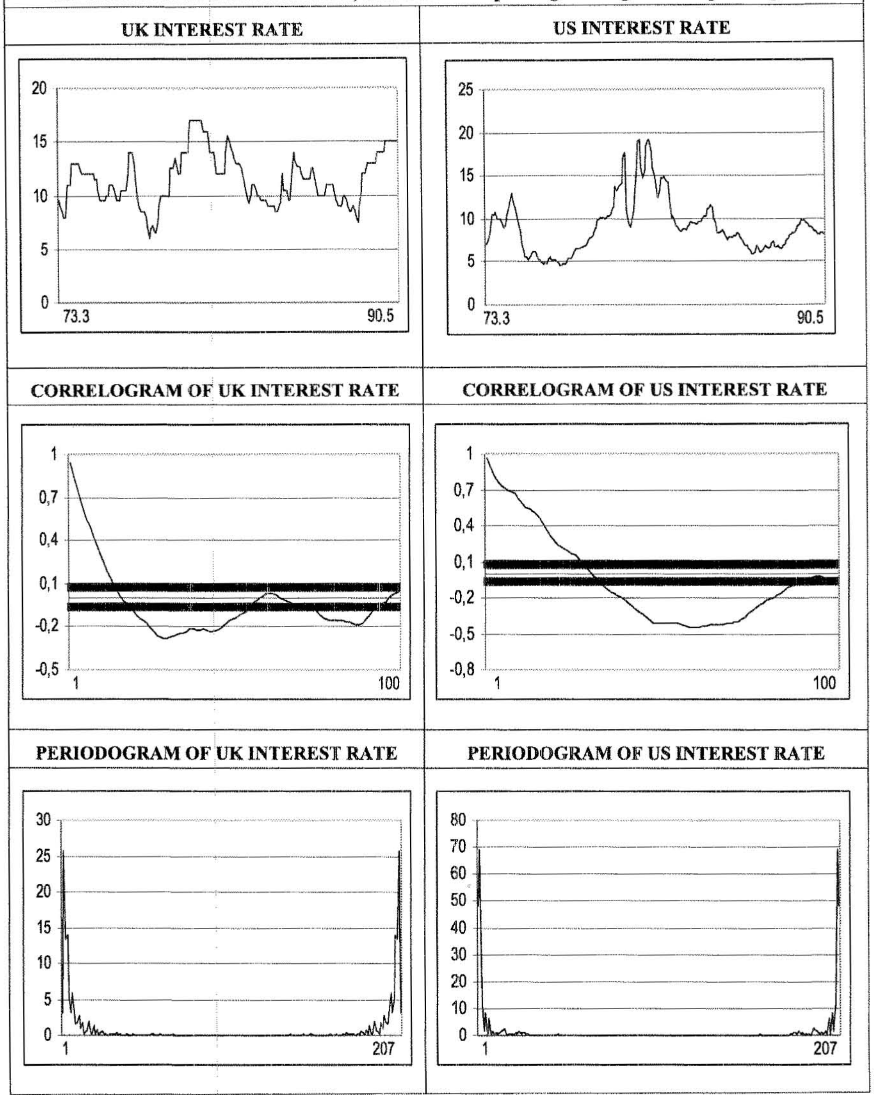
Figure 1 displays plots of the original time series with their corresponding correlograms and periodograms. We see through the correlograms that the values decrease very slowly suggesting the nonstationary nature of the series. Similarly, the periodograms show a large value around the smallest frequency, which may be an indication of long memory behaviour.[1]

Figure 2 displays similar plots for the first differenced data. We see that the time series have now a stationary appearance though the correlograms still show some significant values even at some lags relatively far away from zero which may suggest that the original time series are I(d) with d smaller than or greater than 1. Note, however, that the periodograms do not show evidence of peaks at the smallest frequencies and though they are not consistent estimates of the spectral density, they may suggest that the differenced series are I(0).

[1] The spectral density function of an I(d) process with $d > 0$ has a pole at the 0-frequency. Thus the periodogram should mimick that feature at the smallest frequency.

FIGURE 1

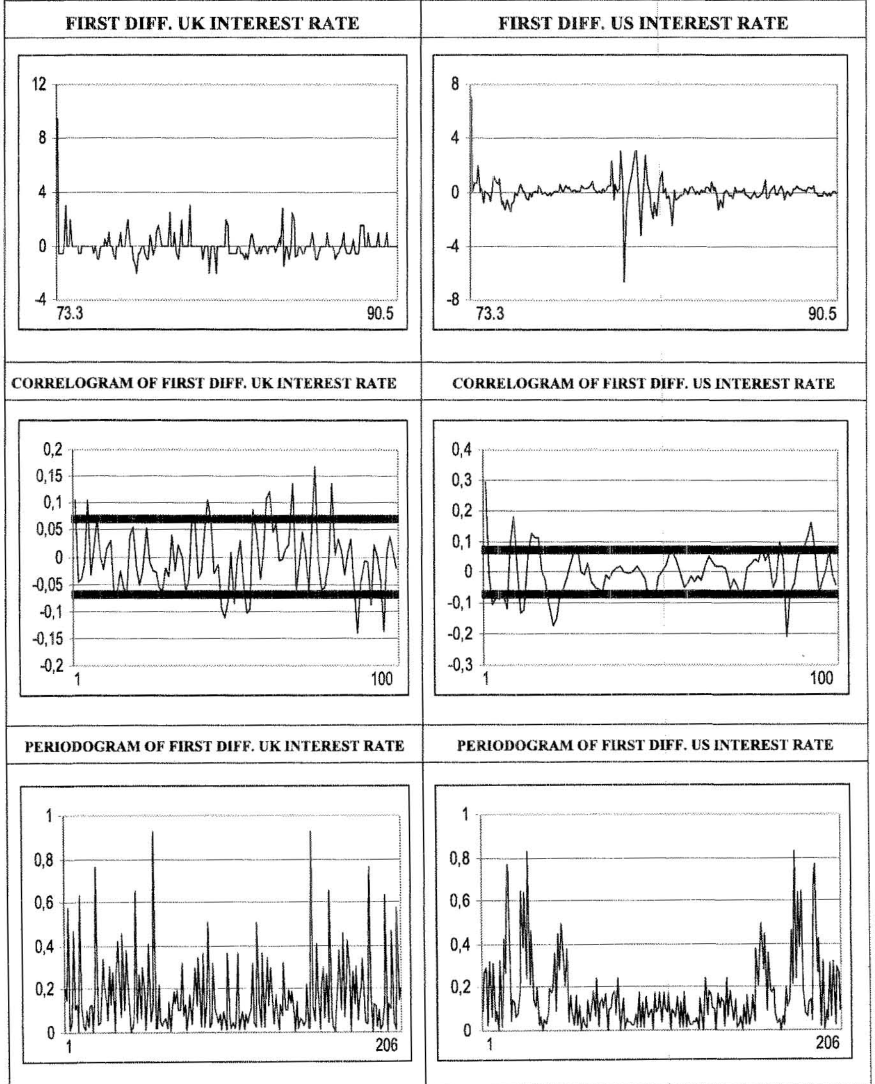
The UK and the US interest rate series, with their corresponding correlograms and periodograms



The large sample standard error under the null hypothesis of no autocorrelation is $1/\sqrt{T}$ or roughly 0.07 for series of length considered here.

FIGURE 2

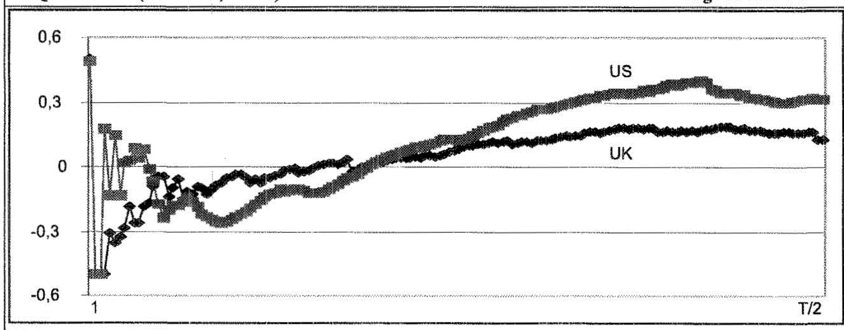
First differences of the UK and the US interest rate series, with their correlograms and periodograms



The large sample standard error under the null hypothesis of no autocorrelation is $1/\sqrt{T}$ or roughly 0.07 for series of length considered here.

FIGURE 3

QMLEs of d (Robinson, 1995a) for the first differences series based on the whole range of values of m



QMLEs of d (Robinson, 1995) for the first differences series based on a shorter range of values of m

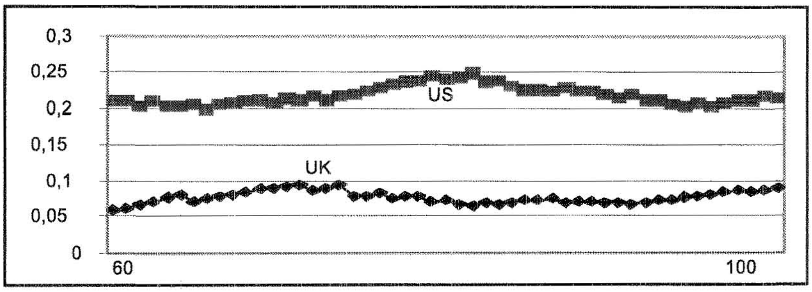


Figure 3 displays the quasi maximum likelihood estimates of d, (i.e., $\hat{d}_1$ given by (3)), based on the first differenced data, using firstly the whole range of values of m from 1 to T/2. We see that these values are very sensitive to m, especially if m is smaller than 50. Thus, in the second plot of the figure, we concentrate on a smaller range of values of m, where the estimates behave relatively stable, in particular, when m is between 60 and 100. We observe that the estimates are in all cases higher than 0, implying that the order of integration of the original series is higher than 1. These values are higher for the US than for the UK, implying a stronger degree of dependence in case of the US interest rate. The estimates oscillate around 0.23 for the US while they are around 0.07 in case of the UK. That means that the orders of integration of the series are approximately 1.23 and 1.07 respectively. In view of these results we can conclude the analysis of these series by saying that both are clearly nonstationary and, what is more important, not mean-reverting, with the effect of the shocks persisting forever.

To corroborate this result, we also performed a simple version of the tests of Robinson (1994a) for testing I(d) statistical models. He proposed a Lagrange Multiplier (LM) test of the null hypothesis:

$$H_o : d = d_o . \tag{4}$$

in (1) for any real value d_o and white noise u_t Specifically, the test statistic is given by:

$$\hat{r} = \frac{T^{1/2}}{\hat{\sigma}^2} \hat{A}^{-1/2} \hat{a} \tag{5}$$

where

$$\hat{a} = \frac{-2\pi}{T}\sum_{j=1}^{T-1}\psi(\lambda_j)I(\lambda_j); \quad \hat{A} = \frac{2}{T}\sum_{j=1}^{T-1}\psi(\lambda_j)^2; \quad \hat{\sigma}^2 = \frac{2\pi}{T}\sum_{j=1}^{T-1}I(\lambda_j); \quad \psi(\lambda_j) = \log\left|2\sin\frac{\lambda_j}{2}\right|.$$

Based on H_o (4), Robinson (1994a) showed that under certain regularity conditions,

$$\hat{r} \rightarrow_d N(0,1) \quad as \quad T \rightarrow \infty, \tag{6}$$

Thus, we are in a classical large-sample testing situation, by reasons described in Robinson (1994a). There also exist more complex versions of the tests (i.e., for example, including autocorrelated disturbances and deterministic regressors) and in all cases, the limit distribution is standard normal. Examples of empirical applications of the tests of Robinson (1994a) based on annual (and also seasonal, quarterly and monthly, and cyclical) data are respectively Gil-Alana (2000), Gil-Alana and Robinson (2001) and Gil-Alana (1999, 2001b).

The test statistic reported in Table 1 is the one-sided one given by (5), so that significant positive values of this are consistent with alternatives of form: H_a: $d > d_o$, whereas significant negative ones imply orders of integration smaller than d_o. In view of this, we should expect a monotonic decrease in the value of the test statistic with respect to d_o, and this is precisely observed across Table 1, where d_o = 0.75, (0.05), 1.50. Starting with the US interest rate, we observe that the unit root null hypothesis, (i.e., d_o = 1), is rejected in favour of higher orders of integration, and the non-rejection values of d always take place when d_o is constrained between 1.05 and 1.35, which is consistent with the earlier discussion based on the QMLE of Robinson (1995a). The results for the UK indicate that the non-rejection values of d occur when d_0 is between 1 and 1.15, again in line with the QMLE. We also extended the analysis to cover the case of weakly parametrically autocorrelated disturbances, in particular, allowing stationary autoregressions for u_t in (1). However, the results in these cases showed a lack of monotonicity in the value of the test statistic with respect to d_0, which may be an indication of potential misspecification. Note that in the event of misspecification, monotonicity is not necessarily to be expected: frequently misspecification inflates both numerator and denominator of $\hat{r}$, to varying degrees, and thus affects to the statistic in a complicated way. Thus, computing $\hat{r}$ for a range of d_0 values may be useful in revealing possible misspecification, though monotonicity is by no means necessarily strong evidence of correct specification. In that respect, however, the results based on the tests of Robinson (1994a) are in line with those obtained with the QMLE, suggesting a stronger degree of association between the observations in the US interest rate compared with the UK case.

TABLE 1

Testing (4) in (1) with the tests of Robinson (1994a)

d	US	UK
0.75	7.667	7.335
0.80	6.365	5.721
0.85	5.185	4.263
0.90	4.126	2.966
0.95	3.178	1.824
1.00	2.332	**0.829'**
1.05	**1.578'**	**-0.033'**
1.10	**0.904'**	**-0.779'**
1.15	**0.301'**	**-1.423'**
1.20	**-0.240'**	-1.979
1.25	**-0.730'**	-2.462
1.30	**-1.174'**	-2.881
1.35	**-1.578'**	-3.248
1.40	-1.948	-3.571
1.45	-2.287	-3.857
1.50	-2.600	-4.111

' and in bold: Non-rejection values of the null hypothesis H_0 (4) at the 95% significance level.

4. Conclusions

We have analysed in this article the monthly structure of the US and the UK interest rates by means of using fractionally integrated techniques. In particular, we were interested in estimating the appropriate order of integration of the series. However, instead of using parametric techniques, which have the problem of being inconsistent in case of model misspecification, we proposed the use of the quasi maximum likelihood estimate of Robinson (1995a). Using this procedure, the results showed that the differencing parameter was higher than 1 for both series, being around 1.23 in case of the US and around 1.07 for the UK. This result was also corroborated when using a simple version of the tests of Robinson (1994a), though with this procedure, the null hypothesis of a unit root was not rejected for the UK.

In view of all this, we can conclude by saying that the interest rates in the US and the UK are both clearly nonstationary. Moreover, the differencing parameter appears to be higher than 1, especially for the US, implying that the degree of dependence between the observations is much stronger for this series than for the UK. The results in this paper also suggest that the traditional approach of taking first differences in the interest rates should be taken with great care in view of the fact that the order of integration appears to be higher than 1. Thus, the differenced series may still have a component of long memory behaviour, with the autocorrelations decaying much slower than expected under the classical I(0) ARMA representations. These results however should be taken with care and a much deeper investigation of interest rates should be addressed to properly understand these series. Thus, for example, the long memory behaviour observed in the differenced US rates could be explained in terms of regime shifts around the mean, the key point being perhaps that the regime shifts never seem to drive interest rates below zero or above 20%, most likely because policy makers do not allow such outcomes in major industrial countries. The possibility of long memory in the context of structural breaks is something that still remains little investigated. Granger and Hyung (1999) and Diebold and Inoue (2001) are among the few works in this area. The tests of Robinson (1994a), briefly described in Section 3, allow us to include deterministic breaks in the model and how the results in these series may be affected by this will be addressed in future papers.

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